Temp. measurement H. T. Hall

A NEW ABSOLUTE NOISE THERMOMETER AT LOW TEMPERATURES[1]

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ABSTRACT

If three resistors, which are kept at different temperatures, are arranged in form of a π network and if two of the thermal noise voltages appearing across the π network are multiplied together and averaged with respect to time, then under certain conditions the correlation between those voltages can be made zero. This condition is used to calculate the temperature of one noise source provided all the resistance values and the other temperatures are known. A noise thermometer of this kind was constructed which is capable of measuring temperatures below approximately 140° K. The boiling points of liquid oxygen and liquid nitrogen were determined absolutely within 0.2%. Between 1.3° K and 4.2° K the thermometer had to be calibrated due to errors arising in the equipment and the measured temperatures were then accurate within ±1%.

I. INTRODUCTION

This paper deals with the construction of a thermometer which makes use of the thermal fluctuations of voltage across an impedance to measure absolutely temperatures below approximately 140° K. Preliminary investigations were carried out for such a device to measure accurately temperatures in the liquid helium region.

According to Nyquist's (1928) law the mean-square voltage fluctuations arising from the thermal agitation of the electrons across an impedance, Z, are given by:

$$\overline{v^2} = 4kT\,\mathrm{Re}[Z]\,p(f, T)\,df, \tag{1}$$

where k is Boltzmann's constant, T the absolute temperature, $\mathrm{Re}[Z]$ is the real part of the complex impedance, Z, $p(f, T)$ the Planck factor, and df the frequency interval in which the measurements are performed. The above formula can be derived from the equipartition law and the second law of thermodynamics and the available noise power is a universal function of the frequency and the absolute temperature (see also Van der Ziel 1954). Equation (1) has also been proved for models which describe the random motion of the electrons in a conductor (Bernamont 1937; Bakker and Heller 1939; Spenke 1939). Nyquist's theorem can also be proved for the one-dimensional form of blackbody radiation (Burgess 1941) which is received by an antenna kept in a sphere at uniform temperature. The thermodynamic method has the merit that it is independent of the mechanism causing the noise.

A number of papers have been published (Lawson and Long 1946; Brown and MacDonald 1946; Gerjuoy and Forrester 1947; Cook, Greenspan, and Wussler 1948) which suggests the possibility of using thermal fluctuations of

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voltage across an impedance to measure low temperatures but they do not propose any practical scheme for a thermometer of this kind and no serious attempt has been reported which indicates that such an experiment has been performed at low temperature.

In 1946 Dicke *et al.* reported a radiometer which measures thermal radiation at microwave frequencies. This method is essentially a commutation comparison technique which compares the unknown noise with that of a standard source. The radiometer has been used for observations of microwave radiation from the sun and the moon and for the measurement of atmospheric absorption at several microwave frequencies. Garrison and Lawson (1949) developed an absolute noise thermometer of the Dicke type to measure high temperatures. A chopper at the input of the amplifier is used to connect alternately the thermometer resistor and a resistor at ambient temperature (standard noise source). The principal limitation of such a switching device for comparison of noise voltages is the variation in contact potential of the chopper. Also the ultimate sensitivity of such a thermometer depends upon the noise-signal-to-amplifier-noise ratio. Aumont and Romand (1954) attempted an improvement of Garrison's and Lawson's thermometer, but the final results have not yet been reported. The National Physical Laboratory (1957) reports also an improved noise thermometer for high temperatures ($\sim$1100° C) based on the switching technique which is capable of comparing noise voltages to 0.05%. Cade (1958) uses an electronic switch instead of a chopper.

To avoid any switching device at the inputs of the amplifier and to make noise measurements virtually independent of the amplifier noise, one can arrange three resistors, which are kept at different temperatures, in form of a π network; and if now two of the thermal noise voltages appearing across the network are multiplied together and averaged with respect to time, then under certain conditions the correlation between those voltages can be made zero. From this condition one can calculate the temperature of one noise source,

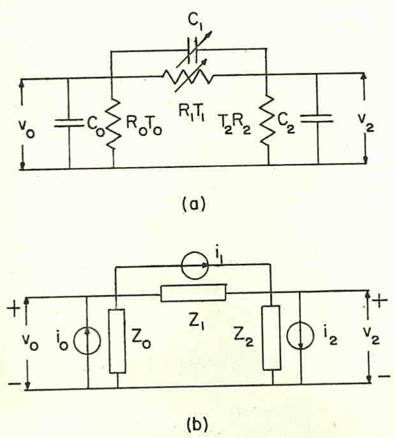

FIG. 1. (*a*) The π network as used to correlate noise between R_0 and R_2 via R_1.
(*b*) The block diagram and the equivalent noise current sources of the π network.

provided all the resistance values and other temperatures are known.[3] Consider the network of Fig. 1. By straightforward network analysis one obtains for v_0 and v_2:

$$v_0 = \frac{Z_0}{Z_T}\,[i_0(Z_1+Z_2)-i_1Z_1-i_2Z_2] \tag{2a}$$

$$v_2 = \frac{Z_2}{Z_T}\,[i_0Z_0+i_1Z_1-i_2(Z_1+Z_0)] \tag{2b}$$

where

$$Z_T = Z_0+Z_1+Z_2, \quad Z_0 = R_0/(1+j\omega C_0R_0), \text{ etc.}$$

and v_0, i_0, etc. are complex vectors. If one multiplies v_0 and v_2 and takes the time average over the product, then one forms:

$$\mathrm{Re}(\overline{v_0v_2^*}) = \mathrm{Re}\left\{\frac{Z_0Z_2^*}{|Z_T|^2}\,[\overline{|i_0|^2}Z_0^*(Z_1+Z_2)+\overline{|i_2|^2}Z_2(Z_0^*+Z_1^*)-\overline{|i_1|^2}|Z_1|^2]\right\}, \tag{3}$$

where

$$\overline{|i_0|^2} = 4kT_0\,df/R_0$$

(the Planck factor is assumed to be unity), similarly $\overline{|i_1|^2}$ and $\overline{|i_2|^2}$. The time average of the products $\mathrm{Re}(\overline{i_0i_1^*})$, etc. are zero because the resistors are independent noise sources. The product $\mathrm{Re}(\overline{v_0v_2^*})$ appearing in equation (3) corresponds to the direct multiplication of the physical voltages v_0 and v_2. From equation (3) one sees that if either $R_0C_0 = R_1C_1 = R_2C_2$ or $(\omega R_nC_n)^2 \ll 1$ the product $\mathrm{Re}(\overline{v_0v_2^*})$ can have a positive or a negative sign provided $T_1>(T_0+T_2)$. For either of the above conditions the value of R_1 required to make $\mathrm{Re}(\overline{v_0v_2^*}) = 0$ can be calculated from equation (3):

$$R_1 = \frac{T_0R_2+T_2R_0}{T_1-T_0-T_2}. \tag{4}$$

In this experiment R_0 and R_2 were both kept in the helium bath so that $T_0 = T_2$. R_2 and R_0 were matched to better than 1/2%, and T_1 was in an isothermal bath at room temperature. If T_1 and the resistances are measured, T_0 can be calculated from

$$T_0 = T_1\frac{R_1}{R_0+2R_1+R_2}. \tag{5}$$

II. THE THERMOMETER AND EXPERIMENTAL PROCEDURES

The first requirement for an absolute noise thermometer of the kind described above is to find some resistors which are stable at liquid helium temperatures, whose values are preferably reproducible for several experiments, which produce no noise in addition to thermal noise, and whose resistive component

[3]This idea was proposed by Dr. J. B. Garrison to Prof. A. W. Lawson of Chicago University (verbal communication by Prof. R. E. Burgess).

is the same as the d-c. resistance (within a specified accuracy) over the frequency interval in which the measurements are performed. Many resistors have been tried at helium temperatures. The ones found most suitable are those manufactured by the Daven Co., series 850. They are hermetically sealed precision metal film type resistors composed of an alloy of pure, noble metals. They are stable over a period of at least 7 hours to better than 1 part in 10^4 and they are reproducible to that accuracy for several experiments. A 20-kΩ resistor has a resistive component of 20kΩ±1% at 3 Mc/sec. The resistance values used in equation (5) should be those appropriate to the frequency range in which the noise measurements are performed. Because no sufficiently accurate audio-frequency bridge was available the metal film deposit resistors were measured at d-c. and at 3 Mc/sec. Since the 3-Mc/sec values differed from the d-c. values by less than 1%, it seems reasonable to conclude that the deviation of the resistance in the audio-frequency range from the d-c. value was less than 0.1% for the above resistors. A 20-kΩ Davohm resistor has a resistance of approximately 17.9 kΩ at liquid helium temperatures (1.3° K to 4.3° K) and the resistance value over this range varies less than 0.05%.

R_1 was a precision wire wound resistance box and C_1 a variable condenser, both kept at room temperature. C_0 and C_2 were the parasitic capacitance between the wires and the shielding, and the input to the amplifiers (including effects due to Miller capacitances), and they were equal within 3%. Unfortunately the parasitic capacity to ground was very large (~220 μμf), and about three-fifths of this was due to the shielding of C_1 and R_1 which was reflected into the input of each amplifier.

Figure 2 shows the block diagram of the thermometer. The shielding requirements of the input circuit and the preamplifiers were very stringent and great care was required to avoid ground loops and to eliminate magnetic pickup in

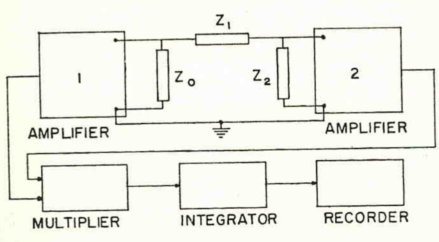

FIG. 2. The block diagram of the noise thermometer.

the π network which in effect acts like a loop. The first tube of the preamplifier was a 6922 (American equivalent to the Philips E88CC) double triode connected as a grounded-cathode-grounded-grid amplifier (cascode) followed by two RC coupled stages (7025 double triode). The cascode and the first RC coupled stage were constructed of wire wound resistors, and their filament currents were supplied by batteries. The lower and upper half power points of the amplifiers were approximately 3 and 7 kc/sec respectively. Both input voltages to the multiplier were constantly monitored by two oscilloscopes and two r.m.s. voltmeters to check the randomness of the noise spectrum and the gain of the amplifiers. The multiplier (a commercial type manufactured by G. A. Philbrick, Inc.) is followed by an amplifier and an RC integrator of variable integration time (22.5 to 90 seconds). The output of the integrator was fed into a recorder via a cathode follower to permit better registration of the time average of the signal.

R_0 and R_2 were carefully cooled to liquid helium temperatures and then their d-c. value was measured with a Wheatstone bridge (R_1 and C_1 were disconnected). R_1 was set to approximately the value at which balance $[\mathrm{Re}(\overline{v_0 v_2^*}) = 0]$ was expected and C_1 adjusted such that $R_1C_1 \simeq R_2C_2$. This was accomplished by connecting Z_1 and Z_2 in series and applying pulses across Z_1 and Z_2. C_1 was then adjusted until the shape of the pulses across the impedance Z_1+Z_2 and Z_2 were identical. This procedure adjusted the time constants τ_1 and τ_2 to approximately 10%. Similarly τ_0 and τ_1 were adjusted and the average setting of C_1 was used when R_1 was varied to achieve balance. If the change in R_1 was large τ_1 had to be rebalanced and R_1 reset. The temperature (room temperature) of R_1 was read on an ordinary mercury thermometer placed on the outside of the resistance box and the value of R_1 was read from the dial setting of the resistance box. The temperature of the helium bath was then calculated from equation (5).

The temperature of the helium bath was sometimes kept constant to better than 1 millidegree K by a temperature regulator (400 c.p.s.) similar to that of Boyle and Brown (1954). A stirrer was sometimes used to equalize the temperature, and the vapor pressure of the helium was measured on a mercury manometer with a cathetometer. A German silver tube (8-mm diameter) extending into the liquid surface was connected to the manometer. The temperature determined from equation (5) was then compared with the "1958 ^{4}He scale of temperatures" (Van Dijk and Durieux 1958; Brickwedde 1958).

III. ERRORS AND LIMITATIONS OF THE THERMOMETER

1. Errors Which Can Be Represented by Noise-Current Sources in Shunt with the π Network

These errors can be divided essentially into two groups: (*a*) errors due to the grid currents, (*b*) errors due to the finite input admittance.

(*a*) The grid current is made up of three parts; electrons arriving at the grid (I_1), electrons emitted from the grid by photoelectric emissions (I_2), and positive ions arriving at the grid (I_3). All three currents are independent of each other. The grid of the triode acts like the anode of a diode; for I_1 it acts like the anode in the exponential part of its characteristic and for I_2 and I_3 it acts like the anode of a saturated diode. Therefore the shot noise due to the grid current is:

$$\overline{i_s^2} = 2e(I_1+I_2+I_3)\,df. \tag{6}$$

The net grid current is $I_g = I_1-I_2-I_3$ and thus $\overline{i_s^2} \geqslant 2eI_g\,df$.

(*b*) The real part of the dynamic input admittance which is a function of frequency consists of three components: the ohmic loss in the input circuit, the cold loss of the first stage (leakage around the bulb of the tube, losses in the socket, etc.), and part of the load of the first stage which is reflected into the input due to the grid-to-plate capacity. In this experiment a cascode input was used which has the advantage that the grounded grid stage reduces the capacitive feedback from output to input without introducing partition noise. However, due to the finite feedback shot noise of the grounded grid stage contributes also to this error.

The errors in (*a*) and (*b*) can be represented by noise-current sources in shunt with the π network. The noise temperature which must be ascribed to this input conductance cannot be determined by calculation because its components and their noisiness are not readily estimated. If one assumes that the real part of the input impedance to the amplifiers is R_g and its effective temperature is $T_g = \alpha T_1$, where α is a constant and T_1 room temperature, and if one also assumes that both amplifiers have the same input characteristics and that $R_g \gg R_0$ and $R_0 \simeq R_2$, then the error in the absolute temperature due to shunt current sources is:

$$\epsilon = \frac{\Delta T}{T} \simeq \frac{R_0}{2kT}\left[e(I_1+I_2+I_3)+2k\frac{\alpha T_1}{R_g}\right] = \frac{A}{T}, \tag{7}$$

where A is a constant for one particular thermometer.

2. Errors Due to Current Flow in the Resistors

Nyquist's law is based upon the assumption that the circuit is a passive network. This requires that no currents are flowing through the resistors R_0, R_1, and R_2. To minimize thermoelectric effects dissimilar materials between the amplifiers and resistors in the network were avoided and the voltage due to this effect was measured to be smaller than 3 μv at the inputs of the amplifiers when R_0 and R_2 were at helium temperatures. The grid current $I_1-(I_2+I_3)$ was approximately 2.6×10^{-9} amperes. Because thin metal layer resistors (R_0 and R_2) consist of a large number of very small conducting particles in loose contact, contact noise may be generated if a current is passed through the resistors. Christenson and Pearson (1936) did not find any contact noise in thin solid carbon filaments when large currents were passed through the specimen. Also Bittel and Scheidhauer (1956) found no noise in addition to the thermal noise when a current was passed through metallic conductors between 45 c.p.s. and 11.5 kc/s. Therefore thin solid metal layer resistors should be free of any noise in excess of thermal noise and Nyquist's law should hold accurately for the above small currents.

3. Non-linearities and Amplifier Noise

Non-linearities in the amplifiers, the multiplier, and the integrator are another source of error. Due to non-linearities the recorder deflection will be increased by an increment proportional to $(|\overline{v_0^2}|\overline{v_2^2}|)^{\frac{1}{2}}$. Because for $Z_1 = \infty$ as well as for balance (see equation 5) the correlation coefficient should be zero for no distortion of the signal, the variance should be the same for both cases. Because both amplifiers were built on different chassis and shielded from each other, the coupling capacity between the amplifiers must have been very small. The zero for balance of the recorder was then determined by grounding the inputs to both amplifiers. Because $|\overline{v_0^2}|$ and $|\overline{v_2^2}|$ are functions of the equivalent noise resistance of the amplifiers, R_n, it is desirable to make R_n as small as possible. R_n for each amplifier was approximately 770 ohms at 300° K over a band width of 3 to 7 kc/s. This was derived by measuringthe recorder deflection (or the squared r.m.s. voltage at the inputs to the multiplier) for $|\overline{v_0^2}|$ and $|\overline{v_2^2}|$ for various input resistances to the amplifiers at room temperature. For a band width of 3 to 12 kc/s the equivalent noise resistance of the amplifiers was 650 ohms. If one assumes that the flicker noise is proportional to $1/f$, then the equivalent noise resistance of the amplifiers at high frequencies is approximately 340 ohms. Therefore, flicker noise was the main contribution to R_n between 3 and 7 kc/s.

4. Errors Due to Mismatch of the Time Constants in the π Network

Equations (4) and (5) were derived under the condition that $(\omega\tau_i)^2 \ll 1$ or that all the τ's are equal. If this does not hold, then equation (4) for $T_0 = T_2$ is modified and one gets:

$$\text{(4a)} \quad \frac{T_0(R_0+R_2)}{T_1-2T_0} = R_1 \frac{1+(\omega^2\tau_0\tau_2)/(T_1-2T_0)\ \{T_1-T_0\tau_1[(1/\tau_0)+(1/\tau_2)]\}}{1+(\omega\tau_1)^2}.$$

If

$$T_1 \gg T_0 \quad \text{and} \quad T_1 \gg T_0\tau_1\left(\frac{1}{\tau_0}+\frac{1}{\tau_2}\right),$$

one obtains:

$$\text{(4b)} \quad \frac{T_0(R_0+R_2)}{T_1-2T_0} \simeq R_1 \frac{1+(\omega\tau_1)^2(\tau_0\tau_2/\tau_1^2)}{1+(\omega\tau_1)^2} = R_1 f(\omega, \tau_i).$$

At helium temperatures it is then sufficient to make $\tau_0\tau_2 \simeq \tau_1^2$. The deviation of $f(\omega, \tau_i)$ from unity will increase with increasing frequency. If both τ_0 and τ_2 are 10% larger than τ_1, then the error at the upper half power frequency is less than 1%. The average error is smaller, because for lower frequencies the error decreases and τ_1 was always adjusted between τ_0 and τ_2. For a systematic error in adjusting τ_1 the fractional error in the noise temperature is almost a constant in the liquid helium range.

5. A-c. Resistance of Thermometer Elements

The deviation of the resistance of R_0, R_1, and R_2 in the audio-frequency range from their d-c. value was estimated to be approximately 0.1% (see above).

6. Response of the Integrator

In the case of a narrow square noise band of width B and uniform spectral intensity a RC integrator of integration time τ will measure with a relative error of a single measurement, β, (Burgess 1951):

$$\text{(8)} \quad \beta = \frac{[\overline{(A-\bar{A})^2}]^{\frac{1}{2}}}{\bar{A}} = (2B\tau)^{-\frac{1}{2}},$$

where $\bar{A}$ is the deflection of the recorder due to the d-c. component of the signal and $\overline{(A-\bar{A})^2}$ the mean-square deviation of the recorder due to the signal.

For $B = 4$ kc/sec and $\beta = 5\times10^{-3}(0.5\%)$ τ should be at least 5 seconds. Integration times from 22.5 to 90 seconds were used. The band widths of both amplifiers were approximately equal.

7. Lead Corrections

If one assumes that the temperature of the leads going to R_0 and R_2 are at room temperature (worst possible case), then the lead resistance should be less than 0.3 ohm for errors smaller than 0.5%, a requirement not difficult to satisfy.

8. Pickup

To avoid errors due to 60 c.p.s. pickup the lower half power points of the amplifiers were designed at approximately 3 kc/sec. Because no shielded room was available experiments could be performed only at night with fluorescent light, thyratron rectifiers, d-c. motors, etc. turned off. Although the amplifiers were protected against shock, audio noise was easily picked up. The voltages were constantly monitored oscillographically at the inputs of the multiplier, to check the randomness of the noise.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Table I shows the boiling points of liquid oxygen and liquid nitrogen measured with approximately 4.48 kΩ metal film deposit resistors (R_0 and R_2) at barometric pressure with an integration time of 90 seconds. They were

TABLE I

Temperatures derived from the vapor pressure, T, measured noise temperatures, T_0, and their ratios for the boiling points of oxygen and nitrogen at barometric pressure

T, °K	T_0, °K(meas.)	T_0/T
90.23	90.26±0.06	1.000±0.001
77.33	77.25±0.08	0.999±0.001

found to be within 0.2% of the temperatures determined from the vapor pressure. The results of the noise–temperature measurements at helium temperatures are shown in Fig. 3. Because, as pointed out in the introduction, the noise power of the real part of an impedance is a universal function of frequency and temperature, any systematic deviation of the noise–temperature can only be due to experimental error of the equipment. The plot in Fig. 3(*a*) can be fitted best by an equation of the form:

$$\frac{T_0}{T} = \frac{A}{T}+b, \tag{9}$$

where $A = 0.385°$ K and $b = 1$ for this thermometer. The term A/T can be explained due to errors which can be represented by noise-current sources in shunt with the π network. Equation (7) shows that this error must be proportional to $1/T$ and the constant of proportionality is $[e(I_1+I_2+I_3)+2k\alpha T_1/R_g]R_0/2k$. If errors due to R_g are neglected, then in the above experi-

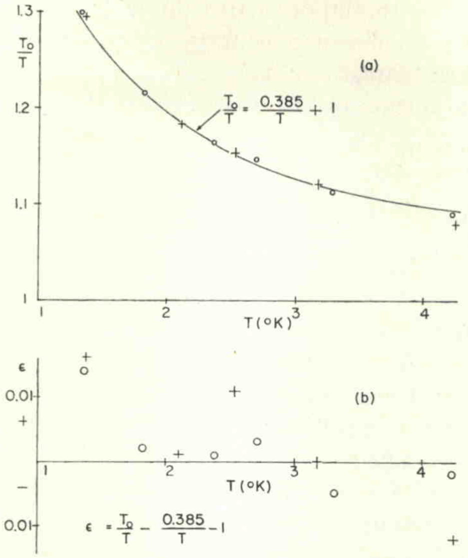

FIG. 3. (*a*) The experimental results at helium temperatures. T_0 is the noise temperature calculated from equation (5) and T the temperature determined from the 1958 helium vapor pressure scale.

(*b*) The deviation ϵ of T_0/T from the equation $A/T+1$ with $A = 0.385°$ K. O represents measurements with an integration time of 90 seconds and + with 67.5 seconds.

ment $I_1+I_2+I_3 = 3.7\times10^{-9}$ amperes. The measured grid current $I_1-(I_2+I_3)$ was approximately 2.6×10^{-9} amperes, which shows that shot noise due to the grid current is the main limitation of the thermometer. At 15 kc/s R_g was measured to be larger than 10^7 ohms, and because the noise temperature $T_g = \alpha T_1$ of R_g is unknown the contribution due to the second term is uncertain. If I_2 and I_3 are neglected compared with I_1 then one can conclude that $R_g/\alpha \simeq 47\times10^6$ ohms.

At helium temperatures equation (4*b*) applies, and over this temperature range a systematic error in adjusting τ_1 gives a fractional error in the noise–temperature which is essentially a constant. This means that b in the equation (9) is not unity. If $\tau_0\tau_2/\tau_1^2 = \text{constant}<1$ for all the measured points between 1.3° K and 4.2° K, the curve in Fig. 3(*a*) will shift down.

In general it will be necessary to measure two known temperatures to calibrate a thermometer of the above kind. These measurements will determine A and b and when T_0 is measured the absolute temperature, T, can be calculated. However, if one is certain that no systematic error is made in balancing the τ's, or if $(\omega\tau_i)^2$ can be neglected with respect to unity then $b = 1$, and only one known temperature is necessary to calibrate the thermometer. Figure 3(*b*) shows the deviation ϵ of T_0/T from the equation $(A/T)+1$ with $A = 0.385°$ K plotted as a function of T. The calibrated thermometer measures temperatures accurately within $\pm1\%$ between 1.3° K and 4.2° K.

This experiment makes use of the correlation of voltages from three independent noise sources at different temperatures to determine the temperature of one (or two) noise sources. This method has the advantage that it eliminates any switching device at the input of the amplifier. The requirements of this method are that for good absolute accuracy of the thermometer the amplifiers, the multiplier, and the integrator must be linear and that $T_1 > (T_0 + T_2)$. At present the main limitation of the absolute accuracy at low temperatures is shot noise generated at the grids of the first stages of the amplifiers. In principle, this method can also be used to measure high temperatures. R_1 could be a fixed resistor at the unknown temperature, and R_0 and R_2 could be kept at room temperature and one or preferably both of them be variable. At high temperatures errors due to shot noise can be neglected. When Z_1 is made infinite and R_0 and R_2 are replaced by two antennas which are located apart from each other, then one has in principle a radio interferometer of the kind developed by Brown and Twiss (1954).

In this experiment it was demonstrated that it is possible and feasible to measure low temperatures absolutely by making use of the thermal fluctuations of voltage across an impedance. Work will continue at this university to improve the accuracy of the noise thermometer, and to derive an absolute temperature scale in the liquid helium region.

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